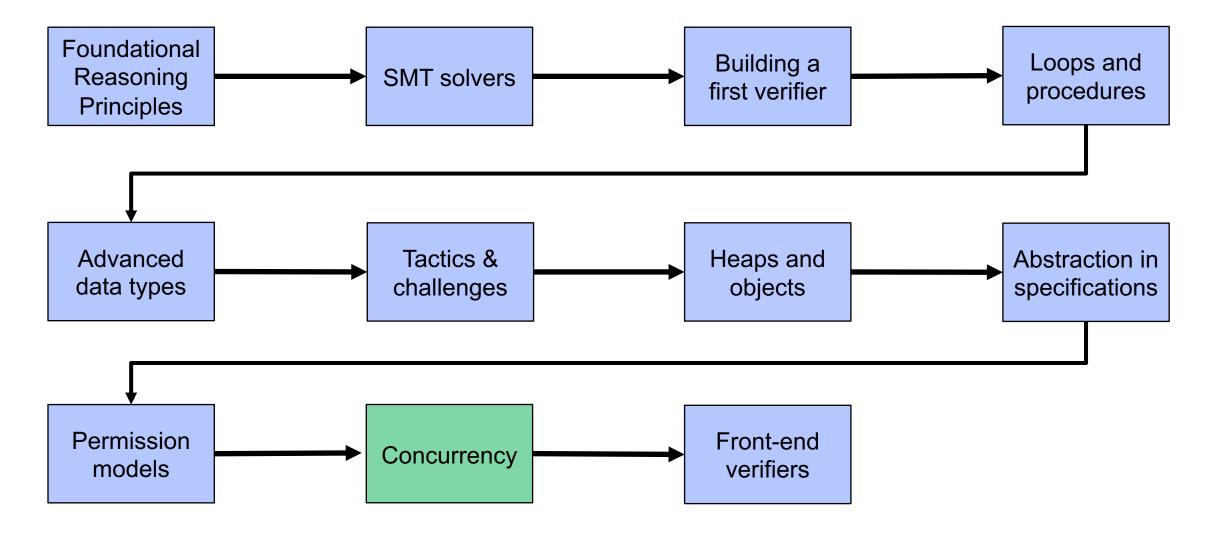
02245 - Module 10

CONCURRENCY



Tentative course outline





But first: the permission story

Who may hold permissions and how are they transferred?



But first: the permission story

Who may hold permissions and how are they transferred?

```
method foo(this: Ref)
  requires acc(this.d)
  ensures acc(this.d)
{
   this.d := 17
}
```

method executions

```
// gain permission
inhale acc(...)
```

```
while (0 < i)
   invariant acc(this.d)
{
   this.d := this.d + i
   i := i - 1
}</pre>
```

loop iterations

```
// give up permission
exhale acc(...)
```

→ Next: method executions may also *run in parallel*

```
predicate list(this: Ref)
{ acc(this.next) && ... }

var x: Ref
x := new(*)
fold(list)
```

predicate instances

```
// trade permission
unfold list(x)
fold list(x)
```

Reasoning about concurrent programs – challenges

Data race: 2+ threads access same data, at least one mutates data

```
acquire x
acquire y
acquire x
...
release x
release y
acquire x
release x
release y
```

Deadlock

```
acquire x
x.f := 5
release x
acquire x
y := 10 / x.f
release x
```

Reasoning about thread interference

Reasoning about thread cooperation

Thread-modular verification

- All verification techniques introduced so far are procedure-modular
 - Reason about calls in terms of the callee's specification
 - Verification of a method does not consider callers or implementation of callees

- We will now present techniques that are also thread-modular
 - Reason about a thread execution without knowing which other threads might run concurrently

```
method create() returns (res: Ref)
  ensures list(res)
  ensures content(res) == Seq[Int]()
{
  ref:= nfl(*)
  res.nex2 := null
  fo daist(res)
}
```

```
acquire x
x.f := 5
release x
acquire x
y := 10 / x.f
release x
```

Both forms of modularity are crucial for verification to scale

Concurrency

1. Concurrency with thread-local state

2. Shared state and synchronization

Thread-local state **a**1 **a**3 a1 := **new**(bal) a2 := new(bal)a3 := **new**(bal) deposit(a2, 150) a1 a1 deposit(a1, 50) transfer(a2, a3, 100) a3 a3 a2 a2 assert a1.bal == a2.bal **a**1 a3

Thread-local state: parallel branches operate on disjoint memory

→ data races are not possible

Structured parallelism

Permissions and separating conjunction lead to a simple proof rule

- All shared memory is on the heap
- Separating conjunction prevents interference between the parallel branches
- Programs with data races have an unsatisfiable precondition

Encoding structured parallelism

The proof rule employs the familiar permission transfer

We can encode this proof rule via exhale and inhale operations

```
method left(...) returns (res1: T)
  requires P1
  ensures Q1
{ // encoding of S1 }
```

Encode left and right branch as methods with specifications

```
exhale P1[...]
exhale P2[...]
havoc res1, res2
inhale Q1[...]
inhale Q2[...]
```

Encode parallel composition like two "half method calls"

```
method busy(courses: Ref, seminars: Ref, exams: Ref, today: Int) returns (res: Bool)
  requires list(courses) && list(seminars) && list(exams)
  ensures list(courses) && list(seminars) && list(exams)
  ensures res == (today in content(courses) | |
                   today in content(seminars) ||
                   today in content(exams))
                                          var rightRes: Bool
                                          rightRes := contains(seminars, today)
 var leftRes: Bool
                                          var res2: Bool
  leftRes := contains(courses, today)
                                          res2 := contains(exams, today)
                                          rightRes := rightRes || res2
                       res := leftRes | rightRes
```

What have we proved when the Viper encoding verifies?

Since contains is side-effect free, concurrent executions should be allowed

```
method getStressed(exams: Ref, today: Int) returns (res: Bool)
  requires list(exams)
  ensures list(exams)
  ensures res == (today in content(exams) | today + 1 in content(exams))
                                            var rightRes: Bool
rightRes := contains(exams, today + 1)
  var leftRes: Bool
  leftRes := contains(exams, today)
  res := leftRes || rightRes
```

- Fractional permissions enable concurrent read access
- but prevent concurrent reads and writes (and, thus, data races)

- In the encoding presented so far, old-expressions in the postconditions of the left and right branch are interpreted incorrectly
- They should refer to the heap before the parallel composition (not the prestate of the enclosing method, which is unsound)

- Viper allows the declaration of labels (at positions where statements may occur)
- Labeled old-expressions are evaluated in the heap at the label
- Encoding of parallel composition uses label to interpret the postconditions of the two branches

```
label branch

// exhale precondition of left block

// exhale precondition of right block

// postcondition of left block

inhale acc(cell1.f) && acc(cell2.f)

inhale cell1.f == old[branch](cell2.f) && cell2.f == old[branch](cell1.f)

// analogous for postcondition of right block
```

Exercise: structured parallelism

- → 04-array-inc-all.vpr
- a. Implement and encode the method below; it increments all elements of an array
- b. Verify memory safety
- c. Specify and verify functional correctness

```
method incrementAll(a: Array)
  requires ...
  ensures ...
{
    ...
    // sequential increment of
    // left half of the array
    ...
}
// sequential increment of
// right half of the array

...
}
```

Solution: structured parallelism

```
method left(a: Array, mid: Int)
  requires arraySeg(a, 0, mid)
  requires 0 <= mid && mid <= len(a)</pre>
 ensures arraySeg(a, 0, mid)
  ensures forall j: Int :: 0 <= j && j < mid ==> lookup(a, j) == old(lookup(a, j)) + 1
 var i: Int := 0
 while(i < mid)</pre>
    invariant arraySeg(a, 0, mid)
    invariant 0 <= i && i <= mid</pre>
    invariant forall j: Int :: 0 <= j && j < i ==> lookup(a, j) == old(lookup(a, j)) + 1
    invariant forall j: Int :: i <= j && j < mid ==> lookup(a, j) == old(lookup(a, j))
    update(a, i, lookup(a, i)+1)
    i := i + 1
```

Analogous for right branch

Solution: structured parallelism (cont'd)

```
method incrementAll(a: Array)
  requires array(a)
  ensures array(a)
  ensures forall j: Int :: 0 <= j && j < len(a) ==> lookup(a, j) == old(lookup(a, j)) + 1
  var mid: Int := len(a) / 2
  label branch
  // precondition of left block
  exhale arraySeg(a, 0, mid) && mid <= len(a)</pre>
  // precondition of right block
  exhale arraySeg(a, mid, len(a)) && 0 <= mid
  // postcondition of left block
  inhale arraySeg(a, 0, mid)
  inhale forall j: Int :: 0 <= j && j < mid ==> lookup(a, j) == old[branch](lookup(a, j)) + 1
  // postcondition of right block
  inhale arraySeg(a, mid, len(a))
  inhale forall j: Int :: mid <= j && j < len(a) ==> lookup(a,j) == old[branch](lookup(a,j))+1
```

Parallel for-loops

Some languages and libraries provide parallel for-loops

```
parallel for i: Int from 0 to len(a)
{ update(a, i, lookup(a, i) + 1) }
```

We can treat such loops as generalized (unbounded) parallel composition

```
body(0) | body(1) | ... | body(len(a) - 1)
```

 For this purpose, we specify the loop body with a pre- and postcondition (instead of a loop invariant)

```
parallel for i: Int from 0 to len(a)
  requires acc(loc(a, i).val)
  ensures acc(loc(a, i).val)
  ensures lookup(a, i) == old(lookup(a, i)) + 1
{ update(a, i, lookup(a, i) + 1) }
```

old refers to pre-state of the loop

Encoding of parallel for-loops

```
→ 05-par-for-loop.vpr
```

```
parallel for i: Int from 0 to len(a)
  requires acc(loc(a, i).val)
  ensures acc(loc(a, i).val)
  ensures lookup(a, i) ==
        old(lookup(a, i)) + 1
{ update(a, i, lookup(a, i) + 1) }
```

Check that loop body satisfies its specification

Intuition for encoding of loop

```
body(0) | body(len(a)-1)
```

```
exhale pre(0) && ... && pre(len(a)-1)
inhale post(0) && ... && post(len(a)-1)
```

Encoding of loop

Unstructured parallelism (threads)

Most programming languages offer unstructured parallelism via threads

- Fork executes a method call in a new thread, returning a thread object
- Join waits for thread to terminate and returns the results of the forked method
- Structured parallelism can easily be encoded via fork and join

```
x := left(...) | y := right(...)
```

```
t1 := fork left(...)
t2 := fork right(...)
x := join t1
y := join t2
```

Challenges of encoding join-operations

- Analogously to structured parallelism, a join inhales the postcondition of the forked method (for instance, to re-gain permissions passed to the forked thread)
- Challenge: how to identify the postcondition to inhale?

```
var t: Thread
if(b) { t := fork left(...) }
else { t := fork right(...) }
join t
```

```
method m(t: Thread)
{
   join t
}
```

Examples use a source language, not Viper

 We assume a type system that parameterizes type Thread with the method that has been forked

```
var t: Thread<left>
if(b) { t := fork left(...) }
else { t := fork right(...) }
join t
```

```
method m(t: Thread<left>)
{
   join t
}
```

Challenges of encoding join-operations (cont'd)

The postcondition of a forked method will in general refer to method parameters

```
method double(p: Int) returns (res: Int)
  ensures res == p + p
```

For a join, the corresponding fork is not statically known

```
var t: Thread<double>
if(b) { t := fork double(5) }
else { t := fork double(7) }
y := join t
assert b ==> y == 10
```

```
method m(t: Thread<double>)
{
   y := join t
   assert y == 10
}
```

 Problem: we cannot determine statically how to substitute actual arguments for formal parameters when inhaling the postcondition during a join

Simplified encoding of fork and join

```
method m(p: T<a>) returns (r: T<b>)
  requires P
  ensures Q
```

Encoding of fork stores method arguments in fields of the thread object

```
t := fork m(5)
```

```
field pArg: T<a>

t := new(pArg)

t.pArg := 5
exhale P[p/5]
```

Encoding of join uses these fields to inhale postcondition

```
y := join t
```

inhale Q[p/t.pArg, r/y]

```
method busy(courses: Ref, seminars: Ref, today: Int) returns (res: Bool)
  requires list(courses) && list(seminars)
  ensures list(courses) && list(seminars)
  ensures res == (today in content(courses) || today in content(seminars))
 var r1: Bool; var r2: Bool
 var t1: Thread<contains>; var t2: Thread<contains>
 t1 := fork contains(courses, today)
 t2 := fork contains(seminars, today)
 r1 := join t1
 r2 := join t2
 res := r1 || r2
```

Repeated joins

Since a join inhales permissions, it is unsound to join the same thread twice

```
join t
join t
assert false
```

inhale acc(t.aArg.f)
inhale acc(t.aArg.f)
assert false

To prevent repeated joins of the same thread, the join operation requires and

consumes a dedicated join-permission

```
t := fork m(5)
```

```
y := join t
```

```
t := new(pArg)
t.pArg := 5
exhale A[p/5]
inhale acc(t.joinable)

exhale acc(t.joinable)
inhale B[p/t.pArg, r/y]
```

Reasoning about heap changes

- Analogously to methods and parallel branches, threads may modify the heap
- Therefore, the postcondition of the forked method may contain old-expressions, which can be encoded via labeled old-expressions

```
fork
t := new(pArg)
t.pArg := x
label l
exhale A[p/5]
inhale acc(t.joinable)
```

```
exhale acc(t.joinable)
inhale ... old[1](t.pArg.f) ...
```

 However, this encoding of join requires that the corresponding fork is statically known and in scope

Reasoning about heap changes (cont'd)

In general, the corresponding fork for a join is not statically known

```
var t: Thread<double>
if(b) { t := fork double(5) }
else { t := fork double(7) }
y := join t
```

```
method m(t: Thread<double>)
{
   y := join t
}
```

 In simple cases, we could evaluate old-expressions when a method is forked and store their values in the thread object (like method parameters)

```
method swap(a: Ref, b: Ref)
  requires acc(a.f) && acc(b.f)
  ensures acc(a.f) && acc(b.f)
  ensures a.f == old(b.f) &&
  b.f == old(a.f)
```

- This is difficult when old-expressions occur under conditionals, contain result variables, or evaluate to unbounded data structures
- We simply omit such postconditions during a join (sound but incomplete)

- a. Encode the method on the right; it computes the height of a binary tree (or -1 if the parameter is null)
- b. Verify memory safety
- c. Specify and verify functional correctness using the depth function from the template

```
method parDepth(this: Ref) returns (res: Int)
  requires ...
  ensures ...
  if(this == null) { res := -1 }
  else {
    var r1: Int; var r2: Int
    var t1: Thread<parDepth>
    var t2: Thread<parDepth>
    unfold tree(this)
    t1 := fork parDepth(this.left)
    t2 := fork parDepth(this.right)
    r1 := join t1
    r2 := join t2
    res := max(r1, r2) + 1
    fold tree(this)
```

Solution: threads

```
method parDepth(this: Ref) returns (res: Int)
  requires this != null ==> tree(this)
 ensures this != null ==> tree(this)
 ensures res == depth(this)
  if(this == null) { res := -1 }
 else {
   var r1: Int; var r2: Int
   var t1: Ref; var t2: Ref
   unfold tree(this)
   // t1 := fork parDepth(this.left)
   t1 := new(thisArg)
   t1.thisArg := this.left
   label f1 // not used here
   exhale this.left != null ==>
                    tree(this.left)
   inhale acc(t1.joinable)
```

```
// analogous for:
// t2 := fork parDepth(this.left)
// r1 := join t1
exhale acc(t1.joinable)
inhale t1.thisArg != null ==>
              tree(t1.thisArg)
inhale r1 == depth(t1.thisArg)
// r2 := join t2
exhale acc(t2.joinable)
inhale t2.thisArg != null ==>
              tree(t2.thisArg)
inhale r2 == depth(t2.thisArg)
res := max(r1, r2) + 1
fold tree(this)
```

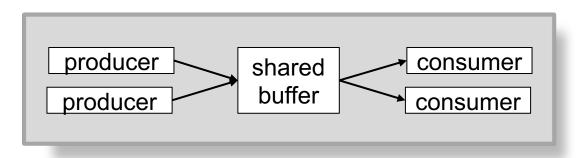
Concurrency

- 1. Concurrency with thread-local state
- 2. Shared state and synchronization

Shared state

- The solution presented so far supports concurrency with thread-local state
- Threads exchange information upon fork and join, but cannot communicate or collaborate while they are running
- Communication between threads is typically supported by shared state or message passing
- We will focus on shared state, but message passing can also be supported using permissions

Example: Producer-Consumer



- Concurrent accesses to mutable shared state require synchronization to prevent data races and ensure correctness
- We will focus on locks as a synchronization primitive

Data race freedom

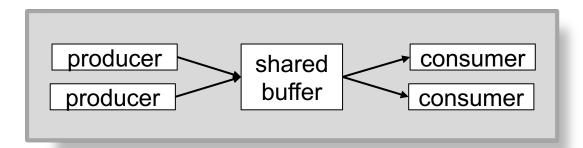
Concurrent accesses to mutable shared state may lead to data races

```
method produce(buf: Ref)
{
    while(true) {
        if(buf.val == null) {
            buf.val := new()
        }
    }
}
```

```
method consume(buf: Ref)
{
    while(true) {
        if(buf.val != null) {
            // consume buf.val
            buf.val := null
        }
    }
}
```

- In verification, permissions can be used to prove the absence of data races (while permitting concurrent reading)
- In programs, synchronization prevents data races

Synchronization via locks



```
method produce(buf: Ref)
{
    while(true) {
        acquire buf
        if(buf.val == null) {
            buf.val := new()
        }
        release buf
    }
}
```

```
method consume(buf: Ref)
{
   while(true) {
      acquire buf
      if(buf.val != null) {
        // consume buf.val
        buf.val := null
      }
      release buf
   }
}
```

- Permission to access buf.val cannot be obtained via the preconditions (that would prevent concurrent executions)
- Permissions transfer happens when acquiring or releasing a lock

Lock invariants

A lock guards accesses to certain memory locations

```
class Buffer {
   @GuardedBy("this")
   Product val;
}
```

Java provides annotations to document which locations are guarded by a lock

We associate each lock with a lock invariant

```
class Buffer {
  lock invariant acc(this.val)
  Product val;
}
```

Permissions in the lock invariant express which locations are guarded by the lock

 Intuition: permissions are held by method executions, loop iterations, predicate instances, or locks

Locks and permission transfer

```
class Buffer {
                                 lock invariant acc(this.val)
                                 Product val;
                                            buf
                                                                method consume(buf: Ref)
method produce(buf: Ref)
  while(true) {
                                                                  while(true) {
    acquire buf
                                                                    acquire buf
    if(buf.val == null) {
                                                                    if(buf.val != null) {
                              buf
                                                         buf
                                                                      // consume buf.val
      buf.val := new()
                                                                      buf.val := null
    release buf
                                                                    release buf
```

More on lock invariants

- A lock invariant holds whenever the lock is not currently held by a thread
- Lock invariants contain arbitrary self-framing assertions

```
acc(this.val) && 0 < this.val

acc(this.val, 1/2)

forall x: Ref :: x in s ==> acc(x.val)
```

Self-framingness is crucial for soundness

```
Ø < this.val</p>
Methods could violate the invariant without acquiring the lock
```

Initializing locks

 Before the first acquire, the lock needs to be initialized, to establish the lock invariant and to transfer the permissions to the lock

```
buf := new(val)
acquire buf // should be rejected
assert false
```

We introduce a ghost statement share that initializes the lock

```
buf := new(val)
share buf
acquire buf // allowed
```

Simplified encoding of locks

- Locks are encoded as references
- To track whether a lock has been initialized, we use the permission to a ghost field isLock

```
share x
```

```
exhale Inv(x)
inhale acc(x.isLock, wildcard)
```

Inv(x) denotes the lock invariant

- Some fractional permission for this field is required to acquire the lock
 - Permission is transferred to each thread that accesses the guarded state
- The rule does not prevent sharing a lock twice
 - Multiple inhales of wildcard are sound

Simplified encoding of locks (cont'd)

- → 09-producer-consumer.vpr
- We model non-reentrant locks (repeated acquire leads to deadlock)
- Therefore, each acquire obtains permissions from the lock

```
acquire x

assert acc(x.isLock, wildcard) permission to acquire is freely duplicable

release x

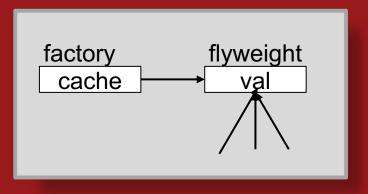
wildcard ensures that the permission to acquire is freely duplicable
```

 The rule for acquire does not prevent deadlock; extra proof obligations can be imposed to ensure that locks are acquired in an order (beyond this course)

Exercise: locking

→ 10-par-flyweight.vpr

Make our previous implementation of the Flyweight pattern thread-safe, that is, use locks to prevent data races



Solution: locking

Make our previous implementation of the Flyweight pattern thread-safe, that is, use locks to prevent data races

```
factory flyweight val
```

```
// the lock invariant is identical
// to the former factory predicate

define Inv(this) (
   acc(this.cache) &&
   (this.cache != null ==>
        acc(this.cache.val, wildcard))
)
```

```
method get(this: Ref) returns (f: Ref)
  requires acc(this.isLock, wildcard)
  ensures acc(f.val, wildcard)
  acquire(this)
  if(this.cache == null) {
      f := new(val)
      this.cache := f
  f := this.cache
  release(this)
```

Client-side vs. server-side locking

Client-side locking

```
method inc(cell: Ref)
  requires acc(cell.val)
  ensures acc(cell.val)
  ensures cell.val == old(cell.val) + 1
{
  cell.val := cell.val + 1
}
```

```
acquire cell
cell.val := 0
inc(cell)
assert cell.val == 1
release cell
```

Server-side locking

```
method inc(cell: Ref)
  requires acc(cell.isLock, wildcard)
  ensures // cannot refer to cell.val
{
  acquire cell
  cell.val := cell.val + 1
  release cell
}
```

```
acquire cell
cell.val := 0
release cell
inc(cell)
acquire cell
assert cell.val == 1
release cell
```

Reasoning about server-side locking

→ 11-owicki-gries.vpr

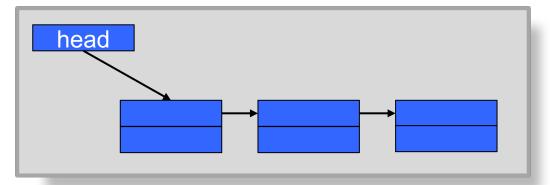
- With server-side locking, methods can typically not provide strong postconditions over the shared data because the permission is not held in the post-state
- In some cases, we can use ghost state to reason about server-side locking
- In general, reasoning about server-side locking requires Owickie-Gries-style relyguarantee reasoning, which takes into account how all other threads may mutate the shared state (beyond the course)

```
method inc(cell: Ref)
{
   acquire cell
   cell.val := cell.val + 1
   release cell
}
```

```
cell := new(val)
cell.val := 0
share cell
t1 := fork inc(cell)
t2 := fork inc(cell)
join t1
join t2
acquire cell
assert cell.val == 2
release cell
```

Coarse-grained locking

Coarse-grained locking uses one lock for the entire data structure



The lock of the list guards accesses to the list and all nodes

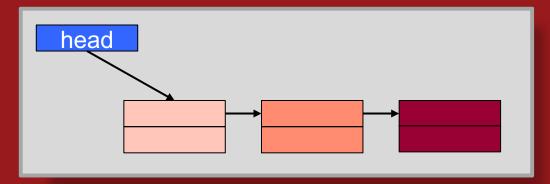
```
method incAll(this: Ref)
  requires acc(this.isLock, wildcard)
{
  acquire this
  if(this.head != null) {
    incAllNodes(this.head)
  }
  release this
}
```

Simple implementation, which uses sequential incAllNodes method, but limits concurrency

Exercise: fine-grained locking

→ 12-fine-grained.vpr

 Fine-grained locking uses multiple locks for the data structure to enable concurrent accesses



Each node has its own lock; multiple threads can traverse the list concurrently

Implement incAll using fine-grained locking

Solution: fine-grained locking

Implement incAll using fine-grained locking

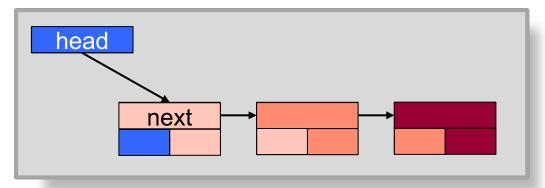
```
define InvList(this) (
   acc(this.head) &&
   (this.head != null ==>
        acc(this.head.isLock, wildcard))
)

define InvNode(this) (
   acc(this.elem) && acc(this.next) &&
   (this.next != null ==>
        acc(this.next.isLock, wildcard))
)
```

```
method incAll(this: Ref)
  requires acc(this.isLock, wildcard)
  acquire this
 var curr: Ref := this.head
 while(curr != null)
    invariant curr != null ==>
         acc(curr.isLock, wildcard)
    acquireNode(curr)
    curr.elem := curr.elem + 1
   var n: Ref := curr.next
   releaseNode(curr)
    curr := n
  release this
```

Fine-grained locking for complex invariants → 13-hand-over-hand.vpr

- Locking nodes in isolation is not possible if the lock invariants relates the states of multiple nodes (for instance, to express that the list is sorted)
- Such examples require hand-over-hand locking



Updating one elem field requires locking two nodes

Summary: concurrency

- We have seen that permissions enable verification for
 - Structured parallelism and threads
 - Data race freedom
 - Share mutable state and locks
- Many additional challenges exist

Properties
deadlock freedom,
starvation freedom,
fairness,
linearizability,
etc.

Implementations
lock-free algorithms,
weak-memory
algorithms,
etc.

Synchronization primitives messages, barriers, etc.

Many of these are active research areas