CONCURRENCY 02245 – Module 10

Tentative course outline

But first: the permission story

■ Who may hold permissions and how are they transferred?

But first: the permission story

■ Who may hold permissions and how are they transferred?

 $→$ Next: method executions may also *run in parallel*

Christoph Matheja – 02245 – Program Verification

Reasoning about concurrent programs – challenges

x.f := x.f + 1 x.f := x.f + 1

Data race: 2+ threads access same data, at least one mutates data

Deadlock

acquire x x.f := 5 **acquire** x **release** x x.f := 0 **acquire** x **release** x y := 10 / x.f **release** x

Reasoning about thread interference

$$
x.f := 0
$$

acquire x
x.f := x.f + 1
release x
acquire x
acquire x
acquire x
assert x.f == 2

x.f := x.f + 1
release x
assert x.f == 2

Reasoning about thread cooperation

Thread-modular verification

- All verification techniques introduced so far are procedure-modular
	- Reason about calls in terms of the callee's specification
	- Verification of a method does not consider callers or implementation of callees

- We will now present techniques that are also thread-modular
	- Reason about a thread execution without knowing which other threads might run concurrently

■ Both forms of modularity are crucial for verification to scale

Concurrency

- 1. Concurrency with thread-local state
- 2. Shared state and synchronization

Thread-local state: parallel branches operate on disjoint memory

 \rightarrow data races are not possible

Structured parallelism

■ Permissions and separating conjunction lead to a simple proof rule

- All shared memory is on the heap
- Separating conjunction prevents interference between the parallel branches
- § Programs with data races have an unsatisfiable precondition

$$
\{\begin{array}{c}\n\text{acc}(x.f) & \text{ } x.f := 7 \{ \ldots \} \\
\text{acc}(x.f) & * \text{acc}(x.f) & \text{ } x.f := 7 \mid y := x.f \{ \ldots \} \\
\end{array}
$$

Encoding structured parallelism

• The proof rule employs the familiar permission transfer

■ We can encode this proof rule via exhale and inhale operations

```
method left(…) returns (res1: T)
   requires P1
   ensures Q1
 \left\{\n \begin{array}{ccc} \n / \n \end{array}\n \right. encoding of S1 }
```
Encode left and right branch as methods with specifications Encode parallel composition

```
exhale P1[…]
exhale P2[…]
havoc res1, res2
inhale Q1[…]
inhale Q2[…]
```
like two "half method calls"

Example: parallel list search

```
\rightarrow 00-busy.vpr
```

```
method busy(courses: Ref, seminars: Ref, exams: Ref, today: Int) returns (res: Bool)
  requires list(courses) && list(seminars) && list(exams)
  ensures list(courses) && list(seminars) && list(exams)
  ensures res == (today in content(courses) || 
                   today in content(seminars) || 
                   today in content(exams))
{
  var leftRes: Bool
  leftRes := contains(courses, today)
                       res := leftRes || rightRes
}
                                          var rightRes: Bool
                                          rightRes := contains(seminars, today)
                                          var res2: Bool
                                          res2 := contains(exams, today)
                                          rightRes := rightRes || res2
```
What have we proved when the Viper encoding verifies?

Example: parallel read access

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■ Since contains is side-effect free, concurrent executions should be allowed

```
method getStressed(exams: Ref, today: Int) returns (res: Bool)
  requires list(exams)
  ensures list(exams)
  ensures res == (today in content(exams) || today + 1 in content(exams))
{
  var leftRes: Bool
  leftRes := contains(exams, today)
  res := leftRes || rightRes
}
                                          var rightRes: Bool
                                          rightRes := contains(exams, today + 1)
```
- § Fractional permissions enable concurrent read access
- but prevent concurrent reads and writes (and, thus, data races)

Parallel branches with side-effects

```
method client(cell1: Ref, cell2: Ref, cell3: Ref, cell4: Ref)
  requires acc(cell1.f) && acc(cell2.f) && acc(cell3.f) && acc(cell4.f) 
{
  cell1.f := 1
  cell2.f := 2swap(cell1, cell2) | swap(cell3, cell4)
  assert cell1.f == 2
}
```
- In the encoding presented so far, old-expressions in the postconditions of the left and right branch are interpreted incorrectly
- § They should refer to the heap before the parallel composition (not the prestate of the enclosing method, which is unsound)

Labeled old-expressions

- Viper allows the declaration of labels (at positions where statements may occur)
- Labeled old-expressions are evaluated in the heap at the label
- Encoding of parallel composition uses label to interpret the postconditions of the two branches

```
label branch
// exhale precondition of left block
// exhale precondition of right block
// postcondition of left block
inhale acc(cell1.f) && acc(cell2.f)
inhale cell1.f == old[branch](cell2.f) && cell2.f == old[branch](cell1.f)
// analogous for postcondition of right block
```
Exercise: structured parallelism

 \rightarrow 04-array-inc-all.vpr

- a. Implement and encode the method below; it increments all elements of an array
- b. Verify memory safety
- c. Specify and verify functional correctness

```
method incrementAll(a: Array)
  requires …
  ensures …
{
  …
  // sequential increment of
  // left half of the array
  …
}
                                           // sequential increment of
                                           // right half of the array
```
Solution: structured parallelism

```
method left(a: Array, mid: Int) 
  requires arraySeg(a, 0, mid)
  requires \theta \leq \text{mid } 88 \text{ mid } \leq \text{ len}(a)ensures arraySeg(a, 0, mid)
  ensures forall j: Int :: 0 <= j && j < mid ==> lookup(a, j) == old(lookup(a, j)) + 1
\{var i: Int := 0
  while(i < mid)
    invariant arraySeg(a, 0, mid)
    invariant 0 <= i && i <= mid
    invariant forall j: Int :: 0 <= j && j < i ==> lookup(a, j) == old(lookup(a, j)) + 1
    invariant forall j: Int :: i <= j && j < mid ==> lookup(a, j) == old(lookup(a, j))
  \{update(a, i, lookup(a, i)+1)
    i := i + 1}
}
```
• Analogous for right branch

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Solution: structured parallelism (cont'd)

 $\{$

}

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```
method incrementAll(a: Array)
  requires array(a)
  ensures array(a)
  ensures forall j: Int :: 0 <= j && j < len(a) ==> lookup(a, j) == old(lookup(a, j)) + 1
  var mid: Int := len(a) / 2
  label branch
  // precondition of left block
  exhale arraySeg(a, 0, mid) && mid <= len(a)
  // precondition of right block
  exhale arraySeg(a, mid, len(a)) && 0 <= mid
  // postcondition of left block
  inhale arraySeg(a, 0, mid)
  inhale forall j: Int :: 0 <= j && j < mid ==> lookup(a, j) == old[branch](lookup(a, j)) + 1
  // postcondition of right block
  inhale arraySeg(a, mid, len(a))
  inhale forall j: Int :: mid <= j && j < len(a) ==> lookup(a,j) == old[branch](lookup(a,j))+1
```
Parallel for-loops

■ Some languages and libraries provide parallel for-loops

```
parallel for i: Int from 0 to len(a) 
{ update(a, i, lookup(a, i) + 1) }
```
We can treat such loops as generalized (unbounded) parallel composition

 $\text{body}(0)$ | $\text{body}(1)$ | \blacksquare | $\text{body}(\text{len}(a) - 1)$

■ For this purpose, we specify the loop body with a pre- and postcondition (instead of a loop invariant)

```
parallel for i: Int from 0 to len(a)
 requires acc(loc(a, i).val)
 ensures acc(loc(a, i).val)
 ensures lookup(a, i) == old(lookup(a, i)) + 1
{ update(a, i, lookup(a, i) + 1) }
```
old refers to pre-state of the loop

Encoding of parallel for-loops

```
parallel for i: Int from 0 to len(a)
  requires acc(loc(a, i).val)
 ensures acc(loc(a, i).val)
 ensures lookup(a, i) == 
           old(lookup(a, i)) + 1
{ update(a, i, lookup(a, i) + 1) }
```

```
method body(i: Int, a: Array) 
  requires \theta \leq i && i \leq len(a)requires acc(loc(a, i).val)
  ensures acc(loc(a, i).val)
  ensures lookup(a, i) == 
             old(lookup(a, i)) + 1
{ update(a, i, lookup(a, i) + 1) }
```
Check that loop body satisfies its specification

Intuition for encoding of loop entitled and the entity of the Encoding of loop

body(0) $\|\cdot\|$ body(len(a)-1)

exhale pre(0) && … && pre(len(a)-1) **inhale** post(0) && … && post(len(a)-1)

```
label l
exhale forall i: Int :: \theta \leq i && i \leq len(a)==> acc(loc(a, i).val) 
inhale forall i: Int :: 0 \le i \& 0 \le len(a)==> acc(loc(a, i).val) 
inhale forall i: Int :: 0 \le i \le \aleph i < len(a)
        ==> lookup(a, i) == old[l](lookup(a, i)) + 1
```
Unstructured parallelism (threads)

■ Most programming languages offer unstructured parallelism via threads

- Fork executes a method call in a new thread, returning a thread object
- Join waits for thread to terminate and returns the results of the forked method
- Structured parallelism can easily be encoded via fork and join

$$
\begin{array}{|l|l|l|}\n \hline\n x := left(...) & y := right(...)\n \end{array}
$$

$$
\begin{array}{rcl}\n\text{t1} &:= & \text{fork left}(\text{...}) \\
\text{t2} &:= & \text{fork right}(\text{...}) \\
\text{x} &:= & \text{join t1} \\
\text{y} &:= & \text{join t2}\n\end{array}
$$

Challenges of encoding join-operations

- Analogously to structured parallelism, a join inhales the postcondition of the forked method (for instance, to re-gain permissions passed to the forked thread)
- Challenge: how to identify the postcondition to inhale?

Examples use a source language, not Viper

■ We assume a type system that parameterizes type Thread with the method that has been forked

Challenges of encoding join-operations (cont'd)

■ The postcondition of a forked method will in general refer to method parameters

```
method double(p: Int) returns (res: Int)
  ensures res == p + p
```
■ For a join, the corresponding fork is not statically known

```
var t: Thread<double>
if(b) { t := fork double(5) }
else { t := fork double(7) }
y := join t
assert b == y == 10
```

```
method m(t: Thread<double>) 
{
  y := join t
  assert y == 10}
```
■ **Problem:** we cannot determine statically how to substitute actual arguments for formal parameters when inhaling the postcondition during a join

Simplified encoding of fork and join

```
method m(p: T<a) returns (r: T<br/>b>)
  requires P
  ensures Q
```
■ Encoding of fork stores method arguments in fields of the thread object

■ Encoding of join uses these fields to inhale postcondition

Example: parallel list search

```
method busy(courses: Ref, seminars: Ref, today: Int) returns (res: Bool)
  requires list(courses) && list(seminars)
  ensures list(courses) && list(seminars)
  ensures res == (today in content(courses) || today in content(seminars))
{
 var r1: Bool; var r2: Bool
 var t1: Thread<contains>; var t2: Thread<contains>
 t1 := fork contains(courses, today)
 t2 := fork contains(seminars, today)
  r1 := join t1
  r2 := join t2
 res := r1 || r2
}
```
Repeated joins

■ Since a join inhales permissions, it is unsound to join the same thread twice

■ To prevent repeated joins of the same thread, the join operation requires and consumes a dedicated join-permission

Reasoning about heap changes

- § Analogously to methods and parallel branches, threads may modify the heap
- § Therefore, the postcondition of the forked method may contain old-expressions, which can be encoded via labeled old-expressions

■ However, this encoding of join requires that the corresponding fork is statically known and in scope

Reasoning about heap changes (cont'd)

In general, the corresponding fork for a join is not statically known

In simple cases, we could evaluate old-expressions when a method is forked and store their values in the thread object (like method parameters)

```
method m(t: Thread<double>) 
\mathcal{A}y := join t
}
```

```
method swap(a: Ref, b: Ref)
 requires acc(a.f) && acc(b.f)
 ensures acc(a.f) && acc(b.f)
 ensures a.f == old(b.f) && 
           b.f = old(a.f)
```
- § This is difficult when old-expressions occur under conditionals, contain result variables, or evaluate to unbounded data structures
- We simply omit such postconditions during a join (sound but incomplete)

Exercise: threads

\rightarrow 08-par-tree-depth.vpr

- a. Encode the method on the right; it computes the height of a binary tree (or -1 if the parameter is null)
- b. Verify memory safety
- c. Specify and verify functional correctness using the depth function from the template

```
method parDepth(this: Ref) returns (res: Int)
  requires …
  ensures …
{
  if(this == null) { res := -1 }else {
    var r1: Int; var r2: Int
    var t1: Thread<parDepth>
    var t2: Thread<parDepth>
    unfold tree(this)
    t1 := fork parDepth(this.left)
    t2 := fork parDepth(this.right)
    r1 := join t1
    r2 := join t2
    res := max(r1, r2) + 1
```

```
fold tree(this)
```
}

}

Solution: threads

```
method parDepth(this: Ref) returns (res: Int)
  requires this != null ==> tree(this)
 ensures this != null ==> tree(this)
 ensures res == depth(this)
{
```

```
if(this == null) { res := -1 }
else {
 var r1: Int; var r2: Int
 var t1: Ref; var t2: Ref
```

```
unfold tree(this)
```
四类

```
// t1 := fork parDepth(this.left)
t1 := new(thisArg)
t1.thisArg := this.left
label f1 // not used here
exhale this. left != left = null ==tree(this.left)
inhale acc(t1.joinable)
```

```
// analogous for:
// t2 := fork parDepth(this.left)
```

```
// r1 := join t1exhale acc(t1.joinable)
inhale t1.thisArg != null ==> 
              tree(t1.thisArg)
inhale r1 == depth(t1.thisArg)
```

```
// r2 := join t2exhale acc(t2.joinable)
inhale t2.thisArg != null ==> 
              tree(t2.thisArg)
inhale r2 == depth(t2.thisArg)
```

```
res := max(n1, r2) + 1fold tree(this)
```
}

}

Concurrency

- 1. Concurrency with thread-local state
- 2. Shared state and synchronization

Shared state

- The solution presented so far supports concurrency with thread-local state
- Threads exchange information upon fork and join, but cannot communicate or collaborate while they are running
- Communication between threads is typically supported by shared state or message passing
- We will focus on shared state, but message passing can also be supported using permissions

§ Example: Producer-Consumer

- § Concurrent accesses to mutable shared state require synchronization to prevent data races and ensure correctness
- We will focus on locks as a synchronization primitive

Data race freedom

§ Concurrent accesses to mutable shared state may lead to data races

- § In verification, permissions can be used to prove the absence of data races (while permitting concurrent reading)
- In programs, synchronization prevents data races

Synchronization via locks

■ Permission to access buf.val cannot be obtained via the preconditions (that would prevent concurrent executions)

■ Permissions transfer happens when acquiring or releasing a lock

}

{

Lock invariants

■ A lock guards accesses to certain memory locations

Java provides annotations to document which locations are guarded by a lock

We associate each lock with a lock invariant

```
class Buffer {
  lock invariant acc(this.val) 
  Product val;
}
```
Permissions in the lock invariant express which locations are guarded by the lock

EXTE: Intuition: permissions are held by method executions, loop iterations, predicate instances, or locks

Locks and permission transfer

More on lock invariants

- § A lock invariant holds whenever the lock is *not* currently held by a thread
- Lock invariants contain arbitrary self-framing assertions

■ Self-framingness is crucial for soundness

0 < this.val Methods could violate the invariant

without acquiring the lock

Initializing locks

■ Before the first acquire, the lock needs to be initialized, to establish the lock invariant and to transfer the permissions to the lock

```
buf := new(val)
acquire buf // should be rejected
assert false
```
■ We introduce a ghost statement share that initializes the lock

Simplified encoding of locks

- Locks are encoded as references
- To track whether a lock has been initialized, we use the permission to a ghost field isLock

- § Some fractional permission for this field is required to acquire the lock
	- Permission is transferred to each thread that accesses the guarded state
- The rule does not prevent sharing a lock twice
	- Multiple inhales of wildcard are sound

Simplified encoding of locks (cont'd)

 \rightarrow 09-producer-consumer.vpr

- We model non-reentrant locks (repeated acquire leads to deadlock)
- § Therefore, each acquire obtains permissions from the lock

■ The rule for acquire does not prevent deadlock; extra proof obligations can be imposed to ensure that locks are acquired in an order (beyond this course)

Exercise: locking

è 10-par-flyweight.vpr

Make our previous implementation of the Flyweight pattern thread-safe, that is, use locks to prevent data races

Solution: locking

Make our previous implementation of the Flyweight pattern thread-safe, that is, use locks to prevent data races

// the lock invariant is identical // to the former factory predicate

```
define Inv(this) (
  acc(this.cache) &&
  (this.cache != null ==> 
        acc(this.cache.val, wildcard))
```

```
method get(this: Ref) returns (f: Ref)
  requires acc(this.isLock, wildcard)
  ensures acc(f.val, wildcard)
{
  acquire(this)
  if(this.cache == null) {
      f := new(val)this.cache := f
  }
  f := this, cacherelease(this)
}
```


)

Client-side vs. server-side locking

Client-side locking and server-side locking client-side locking

```
method inc(cell: Ref)
  requires acc(cell.isLock, wildcard)
  ensures // cannot refer to cell.val
  acquire cell
  cell.val := cell.val + 1release cell
acquire cell
cell.val := 0release cell
inc(cell)
acquire cell
assert cell.val == 1
release cell
```
Reasoning about server-side locking

- With server-side locking, methods can typically not provide strong postconditions over the shared data because the permission is not held in the post-state
- In some cases, we can use ghost state to reason about server-side locking
- In general, reasoning about server-side locking requires Owickie-Gries-style relyguarantee reasoning, which takes into account how all other threads may mutate the shared state (beyond the course)

```
\rightarrow 11-owicki-gries.vpr
```

```
method inc(cell: Ref)
\{acquire cell
  cell.val := cell.val + 1release cell
}
cell := new(val)
cell.val := 0share cell
t1 := fork inc(cell)
t2 := fork inc(cell)
join t1
join t2
acquire cell
assert cell.val == 2
release cell
```
Coarse-grained locking

}

§ Coarse-grained locking uses one lock for the entire data structure

The lock of the list guards accesses to the list and all nodes

```
method incAll(this: Ref)
  requires acc(this.isLock, wildcard)
{
  acquire this
  if(this.head != null) {
      incAllNodes(this.head)
  }
  release this
```
Simple implementation, which uses sequential incAllNodes method, but limits concurrency

Exercise: fine-grained locking

\rightarrow 12-fine-grained.vpr

■ Fine-grained locking uses multiple locks for the data structure to enable concurrent accesses

Each node has its own lock; multiple threads can traverse the list concurrently

■ Implement incAll using fine-grained locking

Solution: fine-grained locking

• Implement incAll using fine-grained locking

```
define InvList(this) (
  acc(this.head) &&
  (this.head != null ==> 
      acc(this.head.isLock, wildcard))
```

```
define InvNode(this) (
 acc(this.elem) && acc(this.next) &&
  (this.next != null =>acc(this.next.isLock, wildcard))
```

```
method incAll(this: Ref)
  requires acc(this.isLock, wildcard)
{
  acquire this
 var curr: Ref := this.head
 while(curr != null) 
    invariant curr != null ==> 
         acc(curr.isLock, wildcard)
  {
    acquireNode(curr)
    curr.elem := curr.elem + 1
   var n: Ref := curr.next
    releaseNode(curr)
    curr := n}
  release this
}
```


)

)

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Fine-grained locking for complex invariants \rightarrow 13-hand-over-hand.vpr

- Locking nodes in isolation is not possible if the lock invariants relates the states of multiple nodes (for instance, to express that the list is sorted)
- Such examples require hand-over-hand locking

Summary: concurrency

- We have seen that permissions enable verification for
	- Structured parallelism and threads
	- Data race freedom
	- Share mutable state and locks
- Many additional challenges exist

Properties deadlock freedom, starvation freedom, fairness, linearizability, etc.

Implementations lock-free algorithms, weak-memory algorithms, etc.

Synchronization primitives messages, barriers, etc.

Many of these are active research areas